

叶轮机粘性流矢量方程在曲线坐标系中展开的新方法和数值计算*

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【摘要】 本文讨论在非正交曲线坐标系中, 粘性流矢量方程一种新的展开思路。应用这种展开方法可以得到项数减少很多的展开式, 便于工程计算。文中同时给出了可压缩与不可压缩方程的展开式, 给出了统一方程表达式。最后对二维不可压叶栅粘性流和三维可压叶栅粘性流进行了数值计算, 计算表明本文给出的新展开方程是有实用价值的。

关键词: 粘性流动 数值计算 叶栅

在叶轮机粘性流场的数值求解中, 由于流场的几何边界极其复杂, 因此必须应用曲线坐标系中的粘性流方程。为了在非正交曲线中计算粘性流, 在方程展开上已经作了不少工作^[1,2], 但上述展开式中均应用张量分量, 方程项数太多, 不便于工程计算。为了解决这一问题, 本文对矢量因变量, 如 $\vec{w}$, 采用笛卡尔分量进行展开, 而坐标架仍采用曲线坐标系的坐标架, 这样作的好处是保持了曲线坐标系的优点, 又应用了笛卡尔分量的简单方便之处, 采用这种思路进行方程展开就得到了方程项数大为减少的展开式。最后应用此展开式对二维不可压叶栅粘性流和三维可压叶栅粘性流进行了数值计算, 计算表明本文给出的新展开方程是有实用价值的。

1 基本方程

非守恒型可压缩时均方程

$$(\partial \rho / \partial t) + \nabla \cdot (\rho \vec{w}) = 0 \quad (1)$$

$$(\partial \vec{w} / \partial t) + \vec{w} \cdot \nabla \vec{w} + 2\vec{\omega} \times \vec{w} - \omega^2 \vec{r} = (-1/\rho) \nabla P + (1/\rho) \nabla \cdot \Pi \quad (2)$$

$$(\partial E / \partial t) + \vec{w} \cdot \nabla E = (1/\rho) \nabla \cdot (P \vec{w}) + (1/\rho) \nabla \cdot (K \nabla T) + (1/\rho) \nabla \cdot (\Pi \cdot \vec{w}) \quad (3)$$

方程中变量 ρ 、 $\vec{w}$ 、 P 、 E 和 T 等为时均量。

非守恒型不可压缩粘流 $\vec{\Omega}$ 、 $\vec{\Psi}$ 、 $\vec{\Omega}$ 、 $\vec{w}$ 时均方程

$$(\partial \vec{\Omega} / \partial t) + \vec{w} \cdot \nabla \vec{\Omega} - \vec{\Omega} \cdot \nabla \vec{w} - 2\vec{\omega} \cdot \nabla \vec{w} = \nu \nabla^2 \vec{\Omega} \quad (4)$$

$$\nabla^2 \vec{w} = -\nabla \times \vec{\Omega} \quad (5)$$

$$\nabla^2 P = -\rho \nabla \vec{w} : (\nabla \vec{w})^T + 2\rho \vec{\omega} \cdot \vec{\Omega} + 3\mu |\vec{\omega}|^2 \quad (6)$$

$$\nabla^2 \vec{\Psi} = -\vec{\Omega} \quad (7)$$

应用积分方程求 P 可用:

$$\nabla \cdot \vec{P} + (\rho/2) W^2] = -\rho(\partial \vec{W}/\partial t) + \rho \vec{W} \times \vec{\Omega} - \mu \nabla \times \vec{\Omega} \quad (8)$$

式中: $\vec{\Omega} = \nabla \times \vec{V} - 2\vec{\omega} = \nabla \times \vec{W}$, $\nu = \mu/\rho$

为了方便推导在非正交曲线坐标系中以笛卡尔分量为因变量的展开式, 首先给出

$$\vec{e}^\alpha = g^{\alpha\beta}(\partial R^y/\partial x^\beta) \vec{i}_y, \quad R^1 = x, \quad R^2 = y, \quad R^3 = z \quad (9)$$

$$T_{\alpha y} = g^{\alpha\beta}(\partial R^y/\partial x^\beta), \quad \vec{e}^\alpha = T_{\alpha y} \vec{i}_y \quad (10)$$

应用关系式(10), 方程(1)~(3)可得展开式:

$$(\partial \rho/\partial t) + T_{\alpha y} \partial(\rho W^y)/\partial x^\alpha = 0 \quad (11)$$

$$\frac{\partial W^y}{\partial t} + T_{\alpha k} W^k \frac{\partial W^y}{\partial x^\alpha} + 2 \in_{mny} \omega^m W^n - \omega^2 r^y = -\frac{1}{\rho} T_{\alpha y} \frac{\partial P}{\partial x^\alpha} + \nu \left[\frac{1}{g} \frac{\partial}{\partial x^\alpha} \left(\overline{g} g^{\alpha\beta} \frac{\partial W^y}{\partial x^\beta} \right) \right]$$

$$y = 1, 2, 3 \quad (12)$$

$$\begin{aligned} \frac{\partial E}{\partial t} + T_{\alpha k} W^k \frac{\partial E}{\partial x^\alpha} = & -\frac{1}{\rho} \frac{1}{g} \frac{\partial}{\partial x^\alpha} \left(\overline{g} P W^\alpha \right) + \frac{1}{\rho} \frac{1}{g} \frac{\partial}{\partial x^\alpha} \left(\overline{g} K' g^{\alpha\beta} \frac{\partial T}{\partial x^\beta} \right) \\ & + \nu \left\{ \frac{1}{g} \frac{\partial}{\partial x^\alpha} \left(\overline{g} g^{\alpha\beta} W^y \frac{\partial W^y}{\partial x^\beta} \right) + T_{\alpha y} \frac{\partial}{\partial x^\alpha} \left(W^\beta \frac{\partial W^y}{\partial x^\beta} \right) \right. \\ & \left. - \frac{2}{3} \frac{1}{g} \frac{\partial}{\partial x^\alpha} \left(W^\alpha \frac{\partial}{\partial x^\beta} \left(\overline{g} W^\beta \right) \right) \right\} \end{aligned} \quad (13)$$

式中, W^y 、 W^k 、 ω^m 、 W^n 均为笛卡尔分量。

方程(11)为 10 项, 增加 6 项, 如果方程(1)用协变分量, 此方程也没有增加项数。方程(12)为 52 项, 比以单变量如协变分量为因变量的运动方程少 148 项, 比以逆变分量的运动方程少 443 项, 其与正交曲线坐标系的方程相差无几。方程(13)项数为 85 项, 统一因变量为 199 项, 比以协变或逆变分量展开的统一变量方程要少 546 项或 558 项。因此, 无论从哪个角度, 方程(11)~(13)的项数是相当少的, 这不仅给工程计算带来方便, 也给书写带来很大好处, 此外因变量也比较直观, 其本身就是物理分量。

对方程(4)、(5)可分别写成:

$$(\partial \Omega/\partial t) + T_{\alpha k} W^k (\partial \Omega/\partial x^\alpha) - (\Omega^k + 2\omega^k) T_{\alpha k} (\partial W^y/\partial x^\alpha) = (\nu/\overline{g}) (\partial/\partial x^\alpha) [\overline{g} g^{\alpha\beta} (\partial \Omega/\partial x^\beta)]$$

$$y = 1, 2, 3 \quad (14)$$

$$(1/\overline{g}) (\partial/\partial x^\alpha) [\overline{g} g^{\alpha\beta} (\partial W^y/\partial x^\beta)] = -\in_{mny} T_{\alpha n} (\partial \Omega^y/\partial x^\alpha) \quad (y = 1, 2, 3) \quad (15)$$

方程(4)、(7)在平面条件下可写成:

$$(\partial \Omega/\partial t) + T_{\alpha k} W^k (\partial \Omega/\partial x^\alpha) = \nu (1/\overline{g}) (\partial/\partial x^\alpha) [\overline{g} g^{\alpha\beta} (\partial \Omega/\partial x^\beta)] \quad (16)$$

$$(1/\overline{g}) (\partial/\partial x^\alpha) [\overline{g} g^{\alpha\beta} (\partial \Psi/\partial x^\beta)] = -\Omega \quad (17)$$

从上面方程看方程项数很少, 方程(14)仅 37 项, 方程(15)仅 11 项, 应用起来非常方便。

2 计算结果和分析

2.1 平面扩压叶栅不可压粘流

本文对一由 NACA 65-(12) 10 叶片组成的平面扩压叶栅进行了计算。方程采用定常条件下的(16)、(17), 湍流模型参阅文献[3]。算例是文献[4]给的实验数据, 计算网格点为 65×97 。进口雷诺数 $Re = 245000$, 在这种高雷诺数条件下, 对流项对粘性影响很大。图 1 是 $\omega = 0.9$ 时

$\Delta\Omega$ 随迭代次数 N 的变化曲线,图2为求解结果,图3为流场6个截面速度分布图,图4为6个截面涡量放大图。从图中可知,粘性只限于一层很薄的粘性较强的边界层内。因此对无分

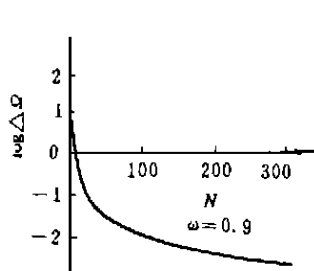


图1 $\Delta\Omega$ 随 N 的变化

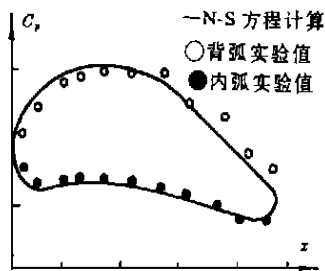


图2 C_p 的变化

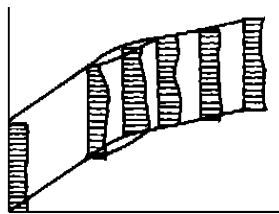


图3 速度分布

离或分离区很小的叶轮机流场用无粘模型或有粘边界层迭代计算是一种很好的方法。另外在尾迹中计算出涡量为零的点,其点连线显然为尾迹线方向,这是用无粘方法很难计算出来的。湍流模型采用等效粘性系数计算,有一些改进。

2.2 分数步长法叶栅三维粘性流场

计算的方程是(11)~(13),湍流模型采用文献[3]。图5为迭代误差收敛过程,从图上看

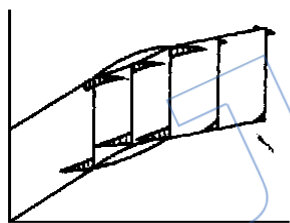


图4 涡量分布

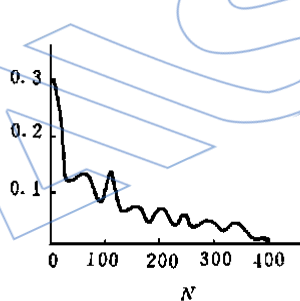


图5 迭代误差收敛过程

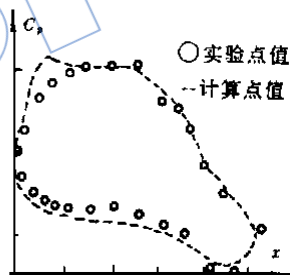


图6 三维计算与实验比较

出,在迭代开始一段过程波动很大,到一定次数就呈收敛。在每秒运算250万次的西门子机上,每次迭代,需要3秒多一些(网格点 $33 \times 17 \times 17$),在细网格上($41 \times 23 \times 23$),每迭代一次要近7秒。图6为三维计算结果与实验比较,直列叶栅中间截面计算值与实验值相近。

参 考 文 献

- 1 汪庆恒,陈乃兴.非正交曲线坐标在叶轮机粘性流动计算中的应用.工程热物理学报,1981,2(4)
- 2 陈乃兴.叶轮机械粘性气动热力学的若干问题—粘性项、传热项及其基本方程的求解途径.中国科学A辑,1983,(10)
- 3 张池平,冯国泰,王民智.可压缩矢量形式湍动能与湍应力方程.哈尔滨工业大学学报,1992,(6)
- 4 Herrig L J, Emery J C, Erwin J R, Systematic Two-Dimensional Cascade Tests of Naca 65-Series Compressor Blades at Low Speeds. NACA TN 3916

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FINITE ELEMENT ANALYSIS AND EXPERIMENTS ON CREEP OF MULTIHOLED PLATE DZ-22

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ABSTRACT

Experiments on tension creep behavior of the ortho-anisotropic material DZ-22 have been performed at 750°C, 850°C and 900°C. This material is applied to directionally-solidified air-cooled turbine blade. A constitutive equation of the creep is formulated by fitting with the experimental data. The equation shows three stages of the creep behavior. Effectiveness of the FEM analysis for the creep behavior of the ortho-anisotropic material is proved by the experimental results. Then, in consideration of the features of the air-cooled blade, a simplified FEM is presented for analysis of the creep of a multiholed component and is proved by the creep experiments with densely multiholed plate specimens.

TRANSIENT BEHAVIOUR OF CRACKED ROTOR THROUGH PARAMETRIC RESONANCE RANGE

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ABSTRACT

In order to diagnose the existence of a crack in rotating machinery, the transient vibration of a cracked rotor through parametric resonance range is investigated. By means of primary harmonic transformation, the normal motion equation of the cracked rotor is transformed into a new differential one. From direct solution of this equation, the amplitude of primary harmonic vibration can be obtained under nonstationary or stationary excitation. In respective consideration of ideal and nonideal sources of energy, it is found that: (1) Whether the source of energy is ideal or not, the largest amplitude of transient vibration near $2\omega/3$ and 2ω (Parametric resonance range) of the rotor with crack is bigger than that without crack (where ω is the critical speed of the rotor without crack). And the deeper the crack, the larger the amplitude in the case of the ratio of crack depth to shaft diameter less than 0.4. (2) When the source of energy is ideal, the larger the acceleration, the less the largest amplitude of transient vibration through parametric resonance ranges of the cracked rotor. Because parametric resonance is the exclusive characteristic of cracked rotor according to the authors' previous work, the authors believe that the transient vibration behaviors near $2\omega/3$ and 2ω may be a basis of diagnosing cracks on a shaft of rotating machinery.

A NEW METHOD FOR EXTENDING VECTOR EQUATIONS OF VISCOUS FLOW IN TURBINE IN CURVILINEAR COORDINATES

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ABSTRACT

A new idea is suggested to extend the viscous vector equations in non-orthogonal curvilinear coordinates. In the expansion equation obtained with the help of this new method, the number of terms is greatly decreased, so that it is easy to apply to engineering calculation. Meanwhile the expanded forms of compressible and incompressible equations are also presented and a generalized expression is given. Finally, by the aid of this new method, the numerical calculations have been carried out for 2-dimensional compressible viscous cascade flow and 3-dimensional compressible viscous cascade flow. It is shown that the new expansion equation is of great value to engineering practice.